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Total no of pages :3
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B.Tech (Sem.1st)

ENGINEERING MATHEMATICS-I
Subject Code :BTAM-101
Paper ID : [A1101]

Time: 3 Hrs. **Max. Marks :60**

Note:- (1) *Question-I is compulsory to attempt, Consisting of ten short answer type question carrying two marks each.*
(2) *Attempt five questions (carrying eight marks each) by electing at least two questions each form Section-A and Section B*

Q1. (a) Graph the set of points whose polar co-ordinates satisfy the conditions.

$r \leq 0, \theta = \pi/4$

(b) Obtain the local extreme values of the function $f(x,y)=x^2+2xy$

(c) Evaluate $\int_0^{\pi}\int_x^{\pi} \frac{\sin y}{y}\, dydx$.

(d) If $f(x,y)=\sum_{n=0}^{\infty}(xy)^n$, given $|xy|<1$, then find $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}$.

(e) $\vec{F}$ and $\vec{G}$ are irrotational vector point functions, then show that $\vec{F} \times \vec{G}$ is a solenoidal function.

(f) State Stoke's Theorem.

(g) If $\vec{F}$ =grad $(x^3+y^3+z^3-3xyz)$,then find curl $\vec{F}$.

(h) Find the work done by the force field $\vec{F} = xy\,\hat{i} + (y-x)\hat{j}$ over the straight line from (1,1) to (2,3).

(i) Rectify the curve $x=a\cos^3 t, y=a\sin^3 t$.

(j) Find the possible percentage error in computing the resistance r from the formula $\frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_2}$, if r_1 and r_2 in error by 2%. (2x10=20)

Section-A

Q2. (a) Trace the curve $x^3+y^3=3axy$ by giving all salient features in detail.

(b) Find the volume of the solid generated by the revolution of the curve $r = a(1+\cos\theta)$ about initial line. (5,3)

Q3. (a) If ρ be the radius of curvature at any point P of the curve $y^2=4ax$ and S is its focus, then show that ρ^2 varies as $(SP)^3$ (5,3)

(b) Find the area between the curve $y^2(2a-x)=x^3$ and its asymptote.

Q4. (a) Find the points on the surface $z^2= xy +1$ nearest to the origin.

(b) Expand $f(x,y)=\tan^{-1} xy$ in ascending powers of $(x-1)$ and $(y-1)$ up to second degree terms. (4,4)

Q5. (a) If $u=\tan^{-1}\frac{x^3+y^3}{x-y}$, then prove that $x^2 \frac{\partial^2 u}{\partial x^2}+2xy \frac{\partial^2 u}{\partial x \partial y}+y^2 \frac{\partial^2 u}{\partial y^2}=2 \sin u \cos 3u$

(b) If $u=\log (x^3+y^3+z^3-3xyz)$, then prove that (4,4)

$$\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)^2 u= -9(x+y+z)^{-2}$$

Section-B

Q6. (a) Using triple integral find the volume of the tetrahedron bounded by the co-ordinate planes and the plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}= 1$

(b) Evaluate : $\int_0^1\int_{x^2}^{2-x} xy\, dxdy$ by changing the order of integration. (4,4)

Q7. (a) Prove that Curl (Curl $\vec{F}$) =grad (div$\vec{F}$) $-\nabla^2$

(b) If $\vec{F}=yz\hat{i} -xz\hat{j} +xy\hat{k}$, then evaluate $\iint_S \vec{F}\cdot\hat{N}\, ds$, where S is the portion of the surface of the sphere $x^2+y^2+z^2=1$,in the first octant. (3,5)

Q8. (a) Apply Stoke's theorem to evaluate the line integral $\int_c [ydx+zdy+xdz]$,

Where C is the curve of intersection of the sphere $x^2+y^2+z^2=a^2$ and the plane $x+z=a$.

(b) Using Green's theorem evaluate the line integral $\int_c[(y-\sin x)dx+\cos x dy]$

Where C is the plane triangle bounded by the lines $y=0,\ x= \pi/2,\ y=(2/\pi)\, x$

Q9. (a) Verify Divergence theorem for $\vec{F}=(x^2-yz)\hat{i}+(y^2-xz)\hat{j}+(z^2-xy)\hat{k}$ Taken over the rectangular parallelepiped $0 \leq x \leq a, 0 \leq y \leq b, 0 \leq z \leq c$.

(b) Evaluate $\iint_R e^{x2+y2}\, dxdy$, where R is semicircular region bounded by the x-axis and the curve $y=\sqrt{1-x^2}$, by changing to polar co-ordinates. (5,3)

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